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AN ANALYTIC AND COMPUTER STUDY OF THE JUMP PHENOMENON IN THE FERRORESONANT REGULATING CIRCUIT*

by

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ABSTRACT

The ferroresonant regulator may exhibit an unexpected phenomenon in its response to amplitude changes in the input voltage. At certain AC input levels, discontinuous jumps appear in the circuit's steady state output vs. input characteristic. This paper derives the conditions under which this action, known as jump resonance, can occur. The ferro is treated as a sinusoidally forced, nonlinear control system with a single nonlinearity. The describing function method of nonlinear system analysis is used to derive the range of circuit parameters in which the jump can occur. It is found that the conditions for the jump can be expressed in terms of the values of the linear energy storage elements and the character of the nonlinearity, and that there exists a unit circle in the Nyquist plane which must be avoided to assure the nonoccurrence of the jump.

The theoretical analysis is supplemented with a computer study of the circuit using the GE Mark II "Analog Computer Simulation" program. Computer calculations of the output voltage at input voltages just above and below the values at which the jump occurs are presented. These indicate the accuracy of the theoretical model.

I. Introduction

The jump phenomenon has been observed in nonlinear control systems for more than twenty years.[1*] Several authors have presented techniques for determining conditions under which "jumps" can occur.[2,3] To most control system engineers, the jump is an undesirable characteristic, and the literature deals mostly with methods of avoiding this mode of operation. On the other hand most ferroresonant regulators inherently will jump into the regulating range as the line voltage is increased, and in fact this may be a desirable mode of operation.

This paper applies techniques of nonlinear control system theory to the ferroresonant regulator and establishes conditions under which the jump can occur. Computer results are also presented.

II. Review of Theory of Ferroresonant Regulators

The ferroresonant regulator is a nonlinear circuit used primarily for stabilizing the half cyclic average value of alternating voltages. The design procedure, assuming a linearization of the nonlinear element, and the theory of operation of the ferro have been discussed in detail.

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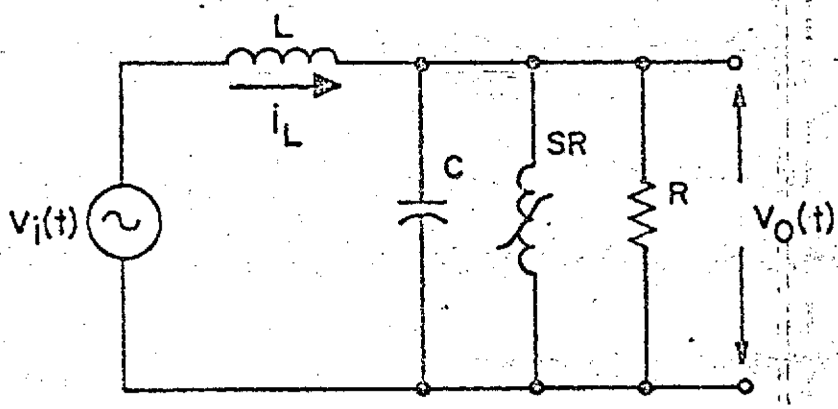
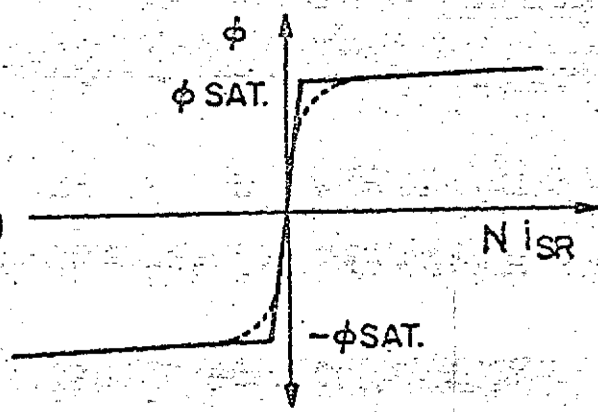
* Superscripts designate references at the end of the text.

Although several forms of the regulator exist, most of them can be reduced to the two-coil (Siemens-Halske) ferroresonant regulator, shown in Figure 1(a). L, C, and R are linear, and SR is nonlinear with a flux-ampere·turn characteristic as shown in Figure 1(b).* Due to the gain



1(a)



1(b)

Figure 1

changing characteristic of SR, the circuit exhibits two very unique properties: regulation and the jump phenomenon. We will now consider the latter.

III. <u>The Control System Model of the Ferroresonant Regulator</u>

Both the state variable[4] and operational model of the ferro will be presented, since the state model is required for the computer simulation, whereas the operational form will be used in the theoretical presentation.

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* Magnetic hysteresis is neglected and an idealized core characteristic is presented. Materials such as oriented 50% ni - 50% iron have this type characteristic. Other materials like grain oriented silicon steel have some "knee" as shown in the dotted lines of Figure 1(b).

Considering the state model, a judicious choice of state variables is the inductor current, i_L, the capacitor voltage, v_o, and the flux in SR, ϕ. The state equations for the circuit of Figure 1(a) are:

$$\frac{di_L}{dt} = -\frac{v_o}{L} + \frac{v_i}{L}$$

$$\frac{dv_o}{dt} = \frac{i_L}{C} - \frac{v_o}{RC} - \frac{f(\phi)}{NC}\ , \quad \text{where } f(\phi) = Ni_{SR}$$

$$\frac{d\phi}{dt} = \frac{v_o}{N} \times 10^{-8}.$$

In matrix form,

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_o \\ \dot{\phi} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{L} & 0 \\ \frac{1}{C} & -\frac{1}{RC} & 0 \\ 0 & \frac{10^8}{N} & 0 \end{bmatrix}}_{\text{linear section}} \begin{bmatrix} i_L \\ v_o \\ \phi \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ -\frac{1}{NC} \\ 0 \end{bmatrix}}_{\text{essential nonlinearity}} f(\phi) + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix}}_{\text{forcing function}} v_i\ .$$

In the above equations i_L, v_o, ϕ, $f(\phi)$ and v_i are functions of time. Note that this choice of state variables require the inverse of the characteristic of Figure 1(b), i.e., $i_{SR} = \frac{1}{N} f(\phi)$.

The control system corresponding to this set of state equations is given in Figure 2, where N.L. represents the nonlinearity f(ϕ).

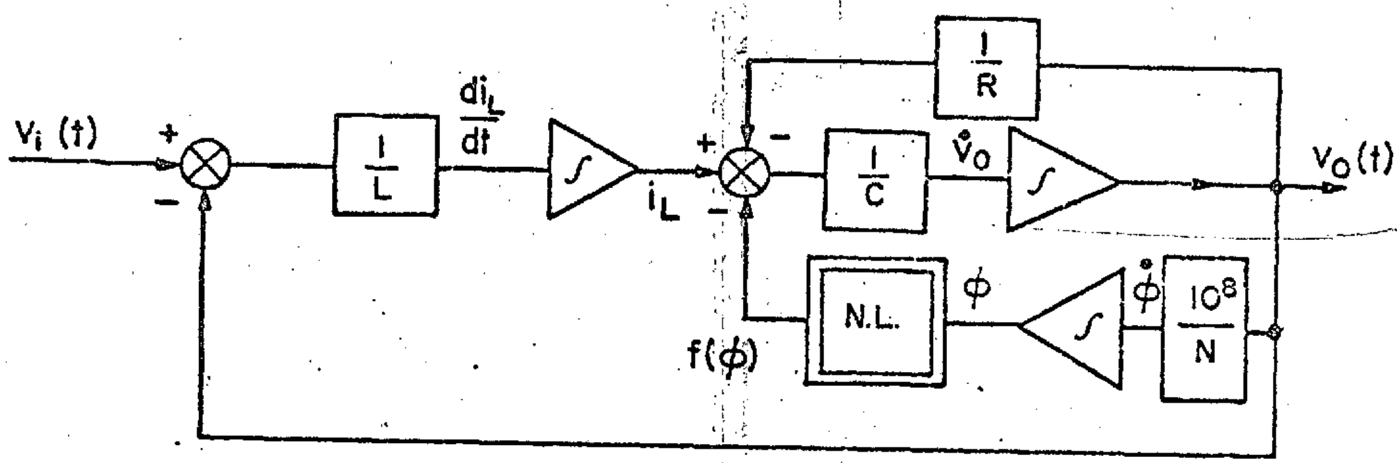


Figure 2

In operational form the control system is that of Figure 3.

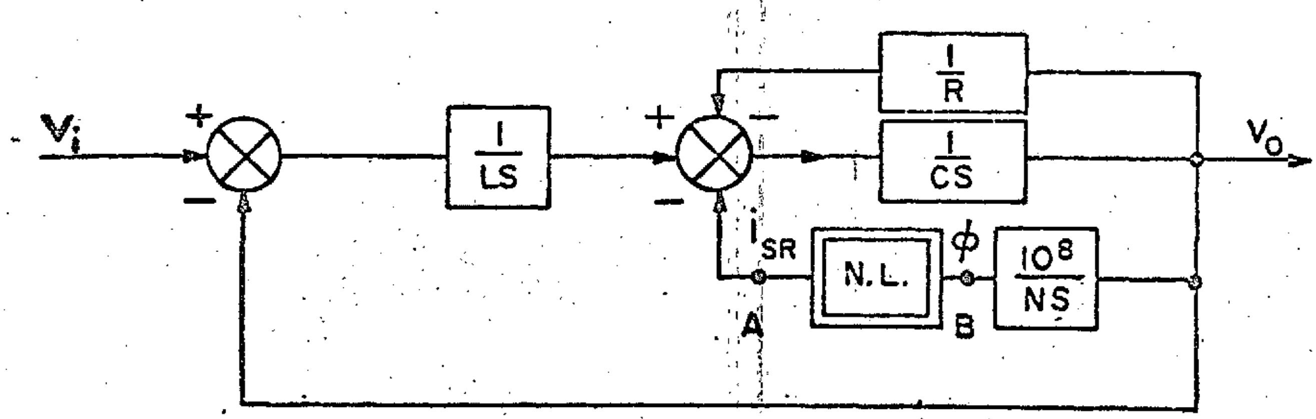


Figure 3

In discussing the jump phenomenon, use will be made of the describing function.[9] The describing function technique is a method of nonlinear system analysis wherein the system is composed of an essential nonlinearity in cascade with a linear plant displaying a low-pass filter characteristic, as in Figure 4. Any harmonics of ω generated by N.L.

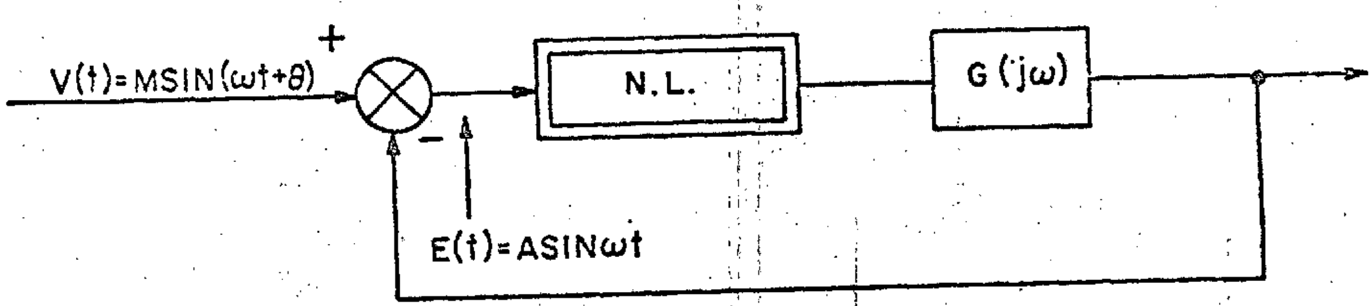


Figure 4

will be filtered by $G(j\omega)$, thus insuring, in effect, a single input frequency to the nonlinearity. In essence the describing function may be treated as a harmonically linearized transfer function. Accordingly the system of Figure 3 can be reduced into the "canonical form" of Figure 4 by noting the signal at "A" and calculating the resulting signal at "B". By this method Figure 3 is reduced to Figure 5.

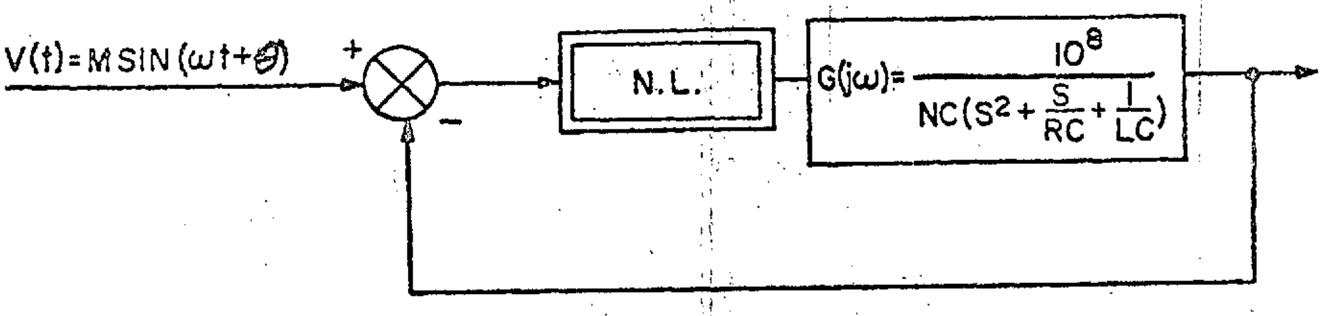


Figure 5

As noted previously, N.L. is now of the form shown in Figure 6(a). In what follows, however, it will be

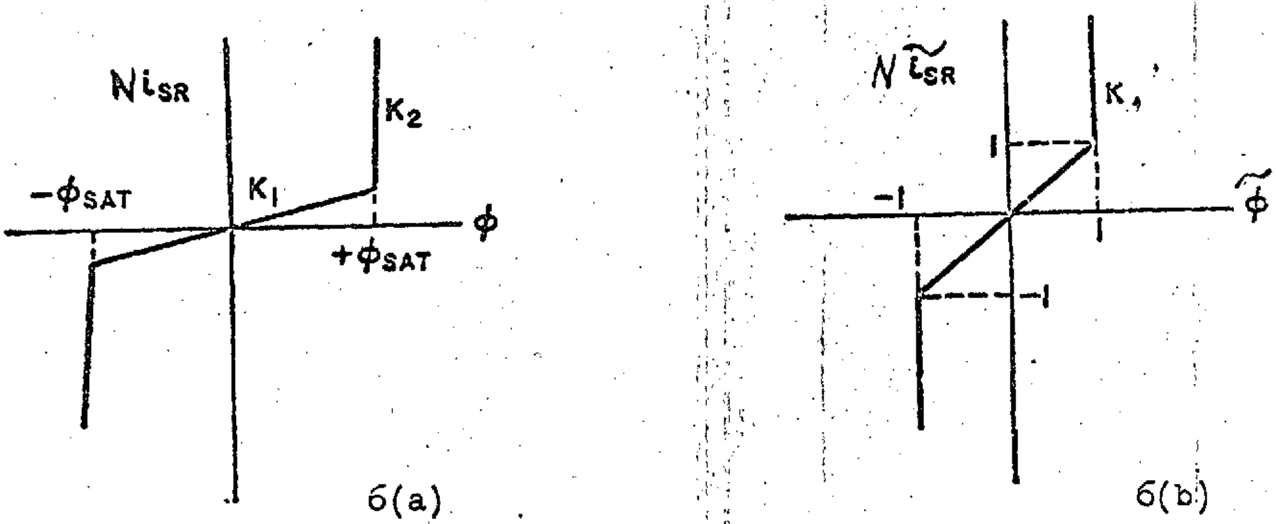
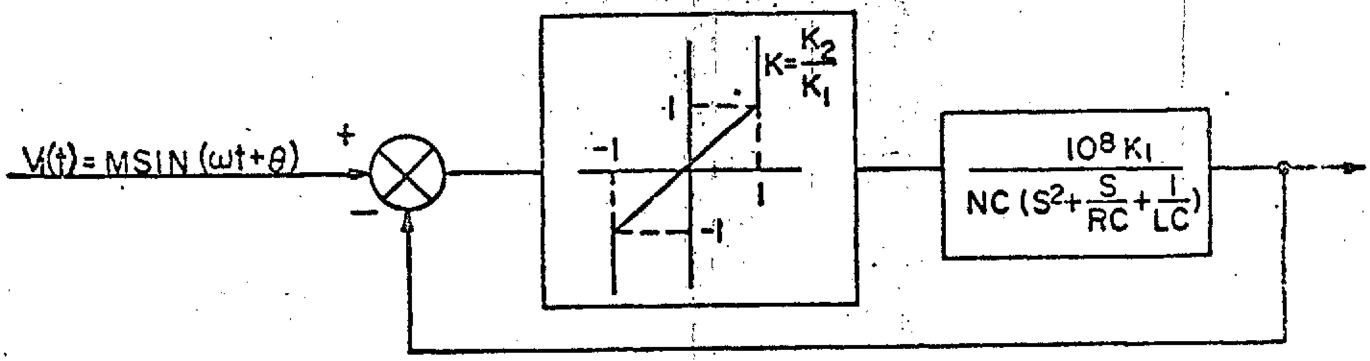


Figure 6

expedient to express the characteristic as in Figure 6(b). If we compare the equations for the respective segments of the figures, it is seen that the final system in canonical form is that of Figure 7. In Figure 7, $V_1(t)$ is a sinusoidal



$$\mathrm{MSIN}(\omega t+\theta) = \frac{|V_i(t)| \times 10^8 \,\mathrm{SIN}\left[\omega t + \left(\mathrm{TAN}^{-1} \dfrac{\omega L}{R(1-\omega^2 LC)} - \dfrac{\pi}{2}\right)\right]}{NLC\, \Phi_{SAT}\, \omega \sqrt{\left(\frac{1}{LC}-\omega^2\right)+\left(\frac{\omega}{RC}\right)^2}}$$

Figure 7

steady state signal, as is the input to the nonlinear element (although of different amplitude and phase). The nonlinear element is shown in normalized form and its describing function is defined as:

$$N(A) = \frac{1}{\pi A}\int_0^{2\pi} f(A \sin \omega t)\sin \omega t \, d\omega t \ .$$

where $f(\cdot)$ is the functional form of N.L. For the particular nonlinearity of Figure 7,

$$N(A) = \begin{cases} 1, & A \leq 1 \\ K - \frac{2}{\pi}(K-1)\left[\sin^{-1}\left(\frac{1}{A}\right) + \frac{1}{A}\sqrt{1-\left(\frac{1}{A}\right)^2}\,\right], & A > 1 \ . \end{cases}$$

IV. Jump Resonance

The following discussion of the jump action refers to the general system shown in Figure 8,* where N.L. is a

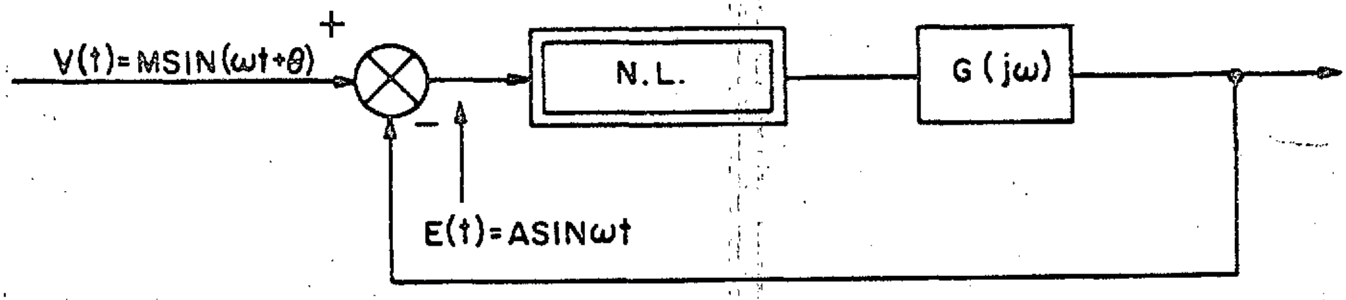


Figure 8

single-valued odd operator of the type in Figure 7 and $G(j\omega)$ is of a sufficiently low-pass nature to allow the use of the describing function. At the jump resonance point, the steady state characteristic of the error amplitude, A, versus the input amplitude, M, becomes discontinuous, as, for example, in Figure 9.

With the foregoing we will now show that there exists a circular region in the Nyquist plane such that if the Nyquist plot of the linear plant intersects this region,

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* The method used here is found in: Hatanaka, H.: The Frequency Responses and Jump-Resonance Phenomena of Nonlinear Feedback Control Systems, Trans. ASME, J. Basic Eng., Vol D-85 (June 1963) pp. 236-242.

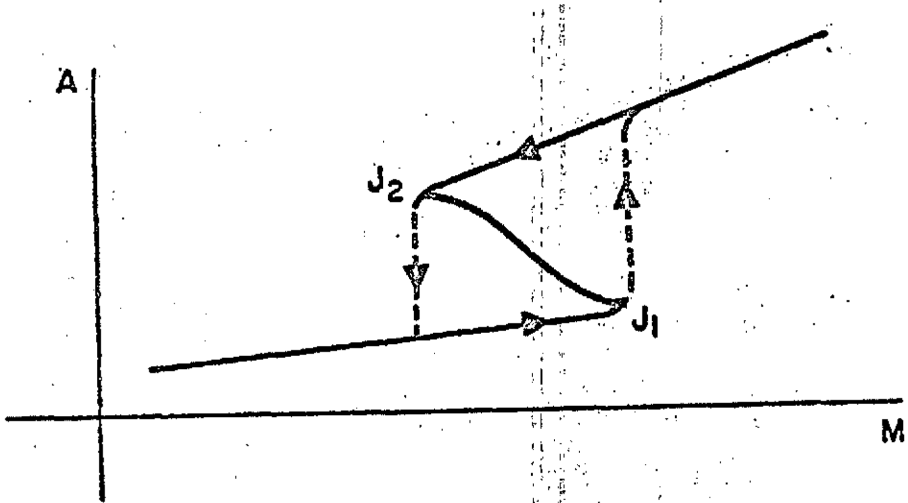

Figure 9

then jump resonance can occur and the input voltage at which it occurs can be determined. If one considers the amplitude M to be a function of A for a fixed frequency, and if the describing function of N.L. is differentiable with respect to A, the system will display jump resonance at the point J1, where evidently the condition:

$\left(\frac{\partial M}{\partial A}\right)_{\omega=\text{const.}} = 0$ is satisfied.

$\left(\frac{\partial M}{\partial A}\right)_{\omega=\text{const.}} < 0$ denotes the region over which the jump can occur (from J1 to J2).

From Figure 8 it is seen that:

$$\frac{E}{V} = \frac{1}{1 + N(A)G(j\omega)}$$

and, therefore,

$$|V| = |E| \times |1 + N(A)G(j\omega)| \text{ , or equivalently,}$$

$$M = A\sqrt{[1 + N(A)U(\omega)]^2 + [N(A)V(\omega)]^2}$$

where

$$G(j\omega) = U(\omega) + jV(\omega)$$

Applying first the condition $\frac{\partial M}{\partial A} = 0$ at the jump point, we see that:

$$\frac{\partial M}{\partial A} = f(U,V,A)$$

$$= \left[U(\omega) + \frac{1}{N(A)}\right]\left[U(\omega) + \frac{1}{N(A) + AN'(A)}\right] + V^2(\omega) = 0 \tag{1}$$

The above equation (in the U,V plane), with A as the parameter, represents a family of circles with centers on the U-axis. Furthermore the inequality $\frac{\partial M}{\partial A} < 0$ represents the interior of these circles.[10]

The envelope of this family of circles must satisfy the relations:[11]

$$f(U,V,A) = 0 \quad \text{and} \quad \frac{\partial f}{\partial A} = 0 \text{ .}$$

Hatanaka shows the solution of these conditions to be:

$$U = -\frac{N'(A)[N(A)+AN'(A)] + N(A)[N(A)+AN'(A)]'}{N'(A)[N(A)+AN'(A)]^2 + [N(A)+AN'(A)]'[N(A)]^2}$$

$$V = \pm\frac{\sqrt{N'(A)[N(A)+AN'(A)]'}\,[N(A)-(AN'(A)+N(A))]}{N'(A)[N(A)+AN'(A)]^2 + [N(A)+AN'(A)][N(A)]^2} \qquad (2)$$

In his referenced article, Hatanaka derives the jump resonance envelopes that satisfy (2) for a gain changing nonlinearity, as shown in Figure 10.

Of particular interest to this discussion are the curves for values of $K > 1$. For the characteristic of Figure 6(b) (Magnetic saturation), K is extremely large; therefor, the contour for $K \to \infty$ may be taken as a limiting case.

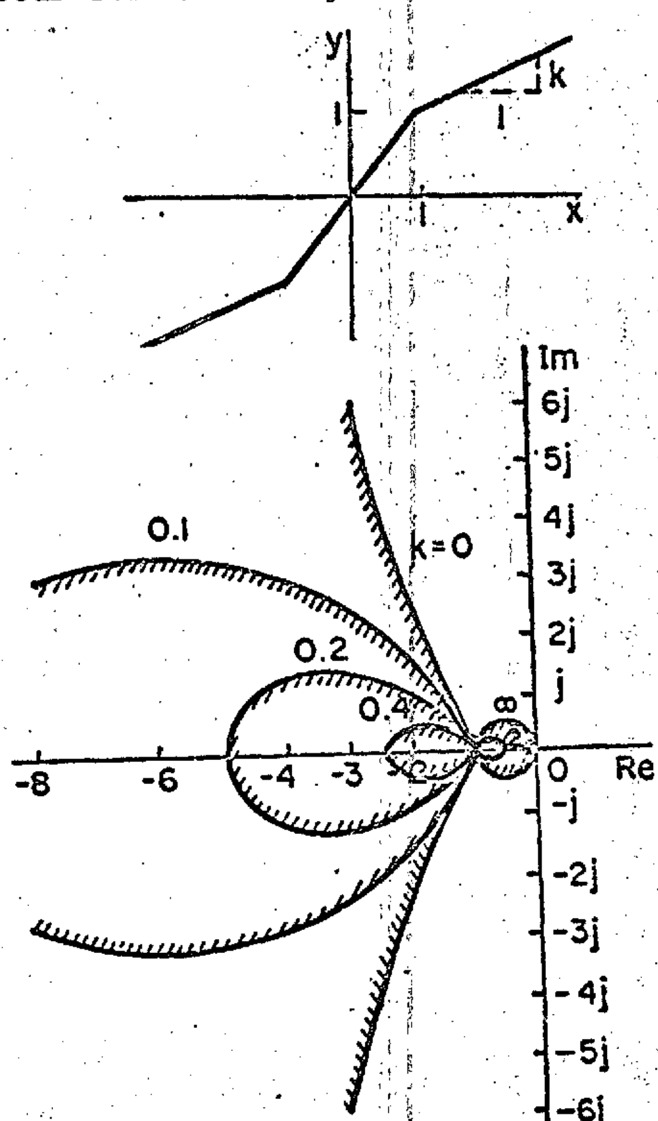

Figure 10

This development leads to the following conclusion: if the polar plot of the linear segment of the system intersects the jump resonance envelope, then that region of intersection denotes the portion of the polar (Nyquist) plane over which jump resonance can occur. In any other region, where the contour is not intersected, jump resonance cannot occur.

In Figure 11 the envelope of interest is repeated along with the Nyquist plot of the linear section of Figure 7. Note, the region in which one finds jumps in the steady state characteristic of $|E|$ versus $|R|$ extends from point X to the origin.

Based on this limiting case of the jump resonance envelope and on the general expression for $G(j\omega)$ one may determine an algebraic criterion for the occurrence or non-occurrence of the jump phenomenon. From Figure 11 we see that if:

$$\left| G(j\omega) + \frac{1}{2} \right| > \frac{1}{2} \tag{3}$$

jumps do not take place.* Utilizing

$$G(j\omega) = \frac{K_1 \times 10^8}{NC} \quad \frac{1}{\left(\frac{1}{LC} - \omega^2\right) + j \frac{\omega}{RC}} , \tag{4}$$

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* It is interesting to note the similarity between this inequality and one obtained by considering Sandberg's circle criterion, which is a sufficiency condition for the stability of nonlinear systems. For example, the nonlinearity discussed above is in the class $A_{1,\infty}$. Therefore the circle, the exterior of which denotes a stable region in the Nyquist plane, is centered at $\left(- \frac{1}{2} , 0 \right)$ with diameter 1. However the circle criterion does not supply any conditions for jump resonance, specifically.

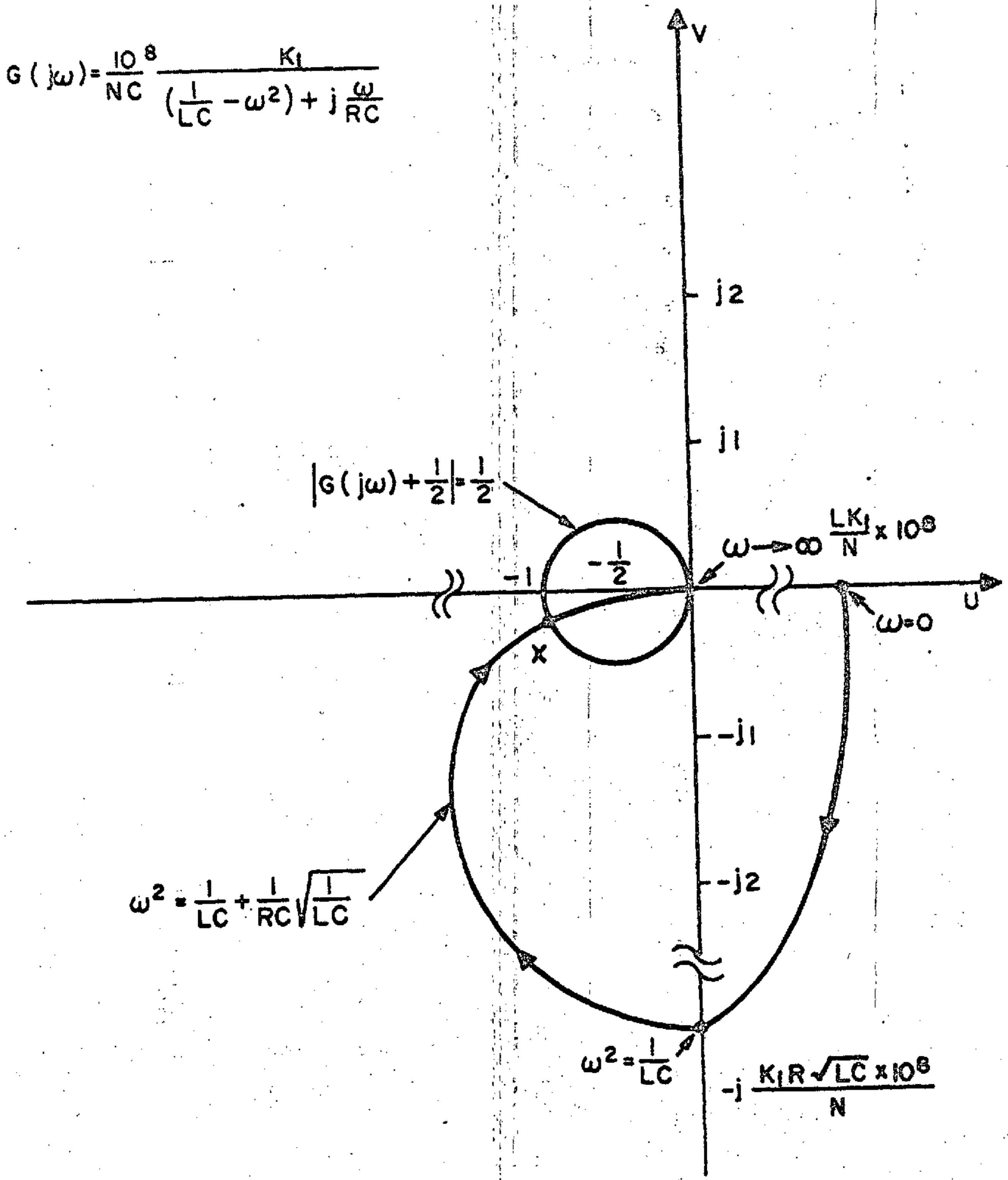

G (jω) = 10^8/NC · K1/((1/LC − ω²) + j ω/RC)
V
j2
j1
|G(jω) + 1/2| = 1/2
ω → ∞ LK1/N x 10^8
−1
−1/2
U
ω=0
X
−j1
−j2
ω² = 1/LC + 1/RC √(1/LC)
ω² = 1/LC
−j K1R √LC x 10^8/N

Figure 11

one determines from equation (3) the relation which will always ensure the nonoccurrence of jump resonance; namely:

$$\omega^2 - \frac{1}{LC} - \frac{K_1 \times 10^8}{NC} < 0 . \qquad (5)$$

Note that this expression indicates that the condition for jump resonance to occur is independent of the load, R.

Substituting equation (4) into equation (1), the input voltage at which the jump occurs can be determined. Here, in general, this voltage will depend on R.

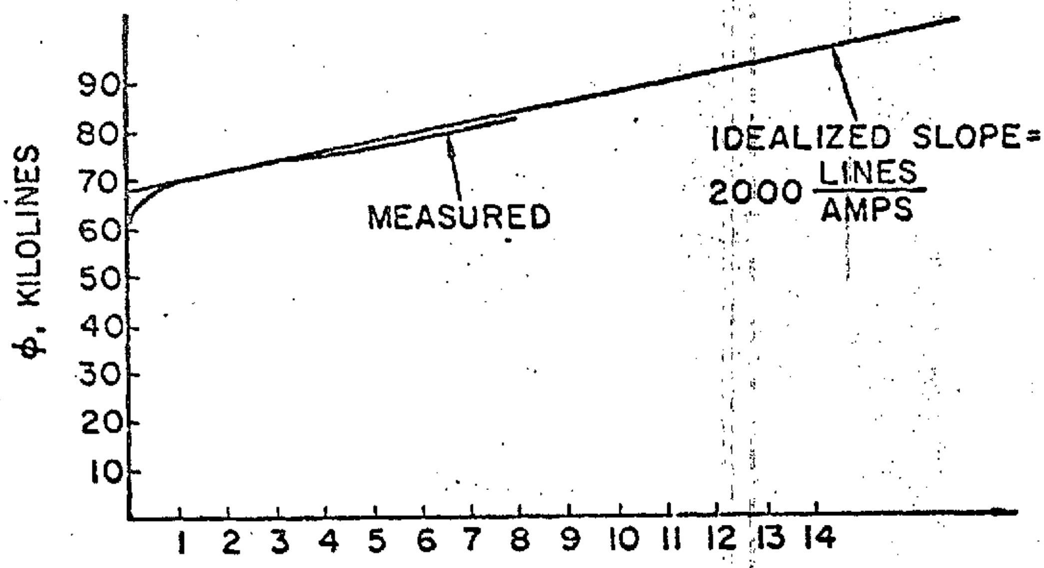
V. Computer Simulation

This part of the paper discusses the computer results of the analysis. The G. E. Mark II Timesharing program "Analog Computer Simulation" was used for the computer study. As its name implies, the program digitally simulates the operation of an analog computer. This program interprets nonprocedural input data for a linear or nonlinear system and solves the resultant integro-differential equation in a state vector formulation. It can be used, as in this case, to analyze a time invariant nonlinear control system. The program will print and/or plot the results. The object of the computer analysis is to show the jump and what occurs while the circuit is going through the jump.

Figure 12 shows a measured ϕ vs. i_{SR} characteristic for a 160 volt, 100 watt regulator. To characterize SR for the "ACS" program, SR is idealized and assumed to have the property

$$i_{SR} = 0, \quad -68{,}000 \leq \phi \leq 68{,}000$$

$$i_{SR} = k\phi \quad \text{for} \quad \phi > 68{,}000 \text{ or } \phi < -68{,}000$$



i_{sr} rms AMPS

Figure 12

The block diagram used for the computer program is given in Figure 13.

From equation (5) the circuit cannot exhibit jump resonance when $\omega^2 - \frac{1}{LC} - \frac{K_1 10^8}{NC} < 0$. For the nonlinear characteristic under investigation, $\frac{K_1 10^8}{NC} = 0$, therefore the jump cannot occur if $\omega L < \frac{1}{\omega C}$.

The program was run for peak input voltages of 149, 184, 240 and 280 $\left(\text{and } \omega L < \frac{1}{\omega C}\right)$. The first four computer plots* characterize the performance at the different input voltages. Note that at the higher inputs, even harmonics being to appear. Further note that even though V_{in} is changing substantially, the half cyclic average value of V_o remains essentially constant and no jump occurs.

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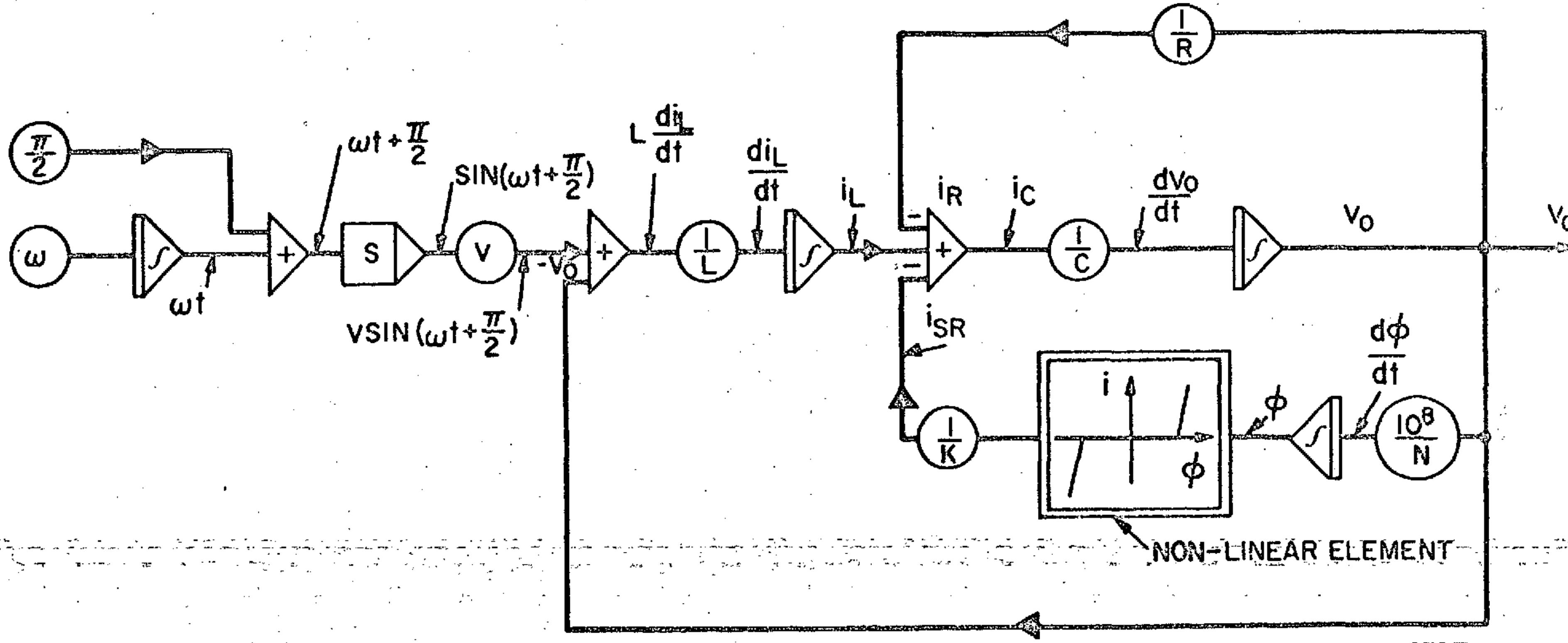
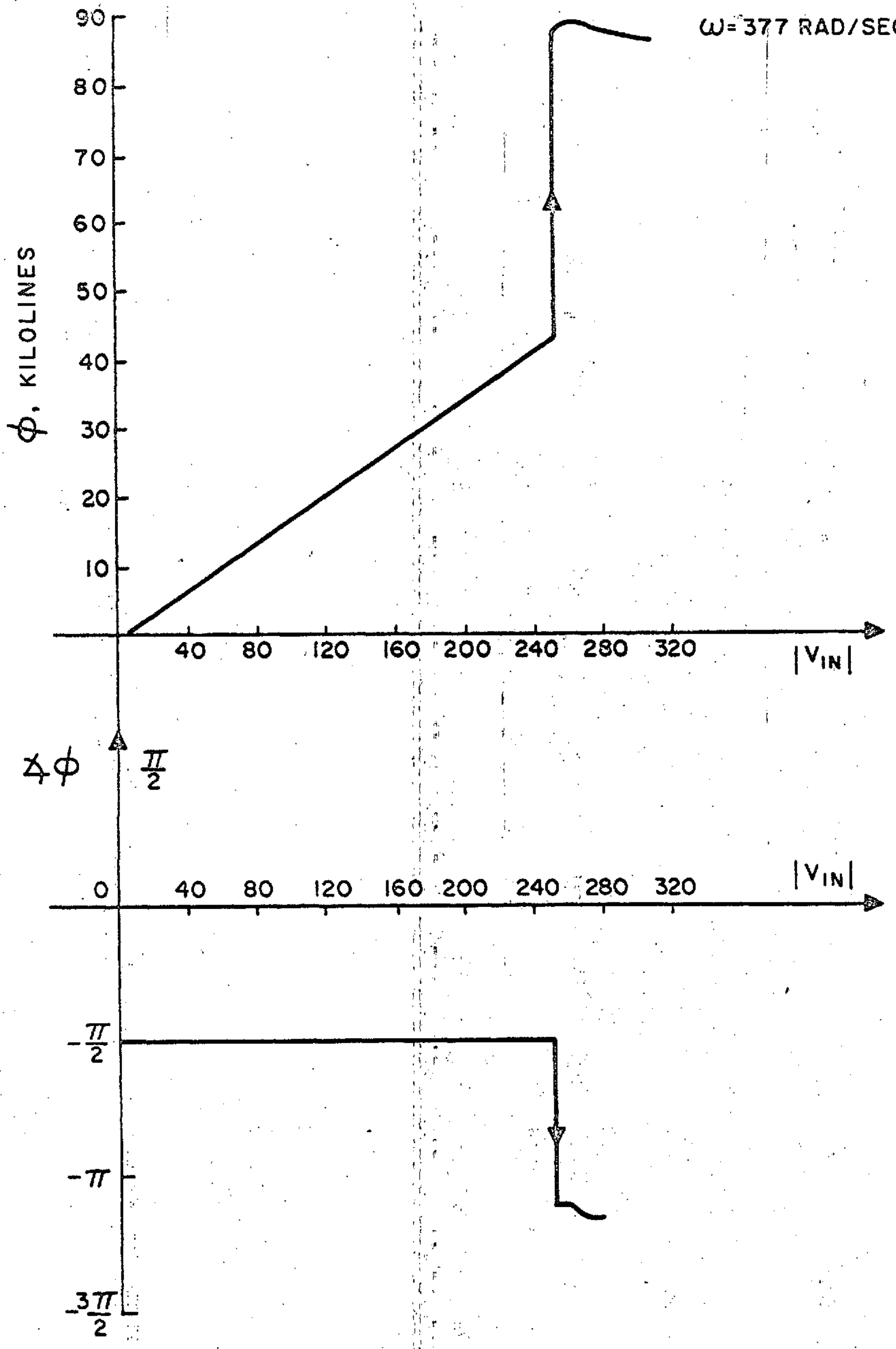
* The computer plots are attached at the end of the memorandum.



π/2
ω
ωt + π/2
SIN(ωt + π/2)
S
V
-Vo
VSIN (ωt + π/2)
ωt
L diL/dt
diL/dt
1/L
iL
iR
iC
1/C
dVo/dt
Vo
Vo
1/R
iSR
1/K
i
φ
φ
dφ/dt
10^8/N
NON-LINEAR ELEMENT

FIGURE 13


ω = 377 RAD/SEC
φ, KILOLINES
90
80
70
60
50
40
30
20
10
40 80 120 160 200 240 280 320
|V_IN|
∡φ
π/2
0
-π/2
-π
-3π/2

The condition $\omega L > \frac{1}{\omega C}$, under which jump resonance can occur, was then investigated. Computer plots are given for input voltages (peak) of 220, 240, 245, 250, 260, and 280. Note that when V_{IN} is increased from 245 to 250 volts, the half cycle average output voltage, the flux, and the phase of the output with respect to the input all exhibit the jump phenomenon. It is interesting to note that at the lower input voltages, before that which precipitates the jump resonance, the core of SR transiently saturates but remains unsaturated in the steady state. See Computer Plots 5, 6, and 7. Figure 14 summarizes the data from the computer simulation under jump conditions.

IV. Conclusion

There are four important results from this study. First, the conditions under which jump resonance can occur in the ferroresonant regulating circuit have been established on an analytical basis. Second, several heretofore empirically observed results, such as the relation between X_L and X_C and the influence of the load have been verified analytically. Third, the methods of nonlinear control theory have been applied to nonlinear magnetics, and finally the Analog Computer Simulation program has been used as a very powerful tool in studying this nonlinear circuit.

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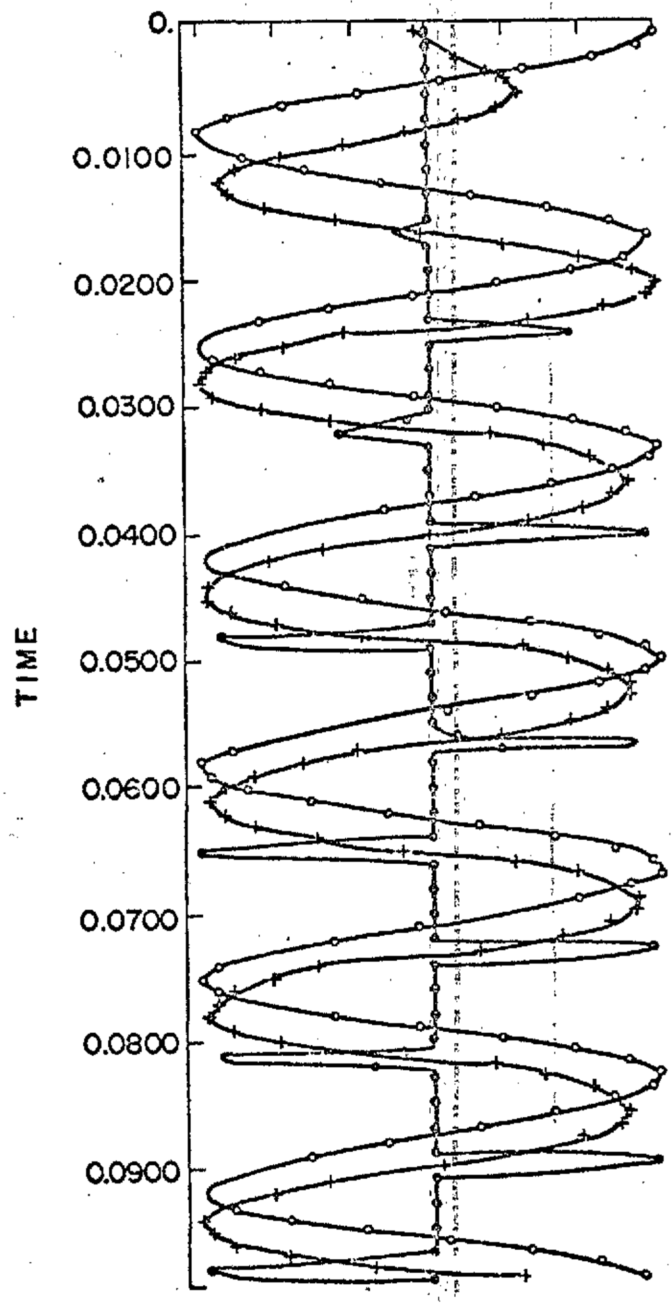
Computer Plot 1

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	+ V_O	−225.90565	246.39653	7.87170
17	∘ i_{SR}	−1.52372	1.52216	0.05076
6	∘ V_I	149.00000	149.00000	4.96667



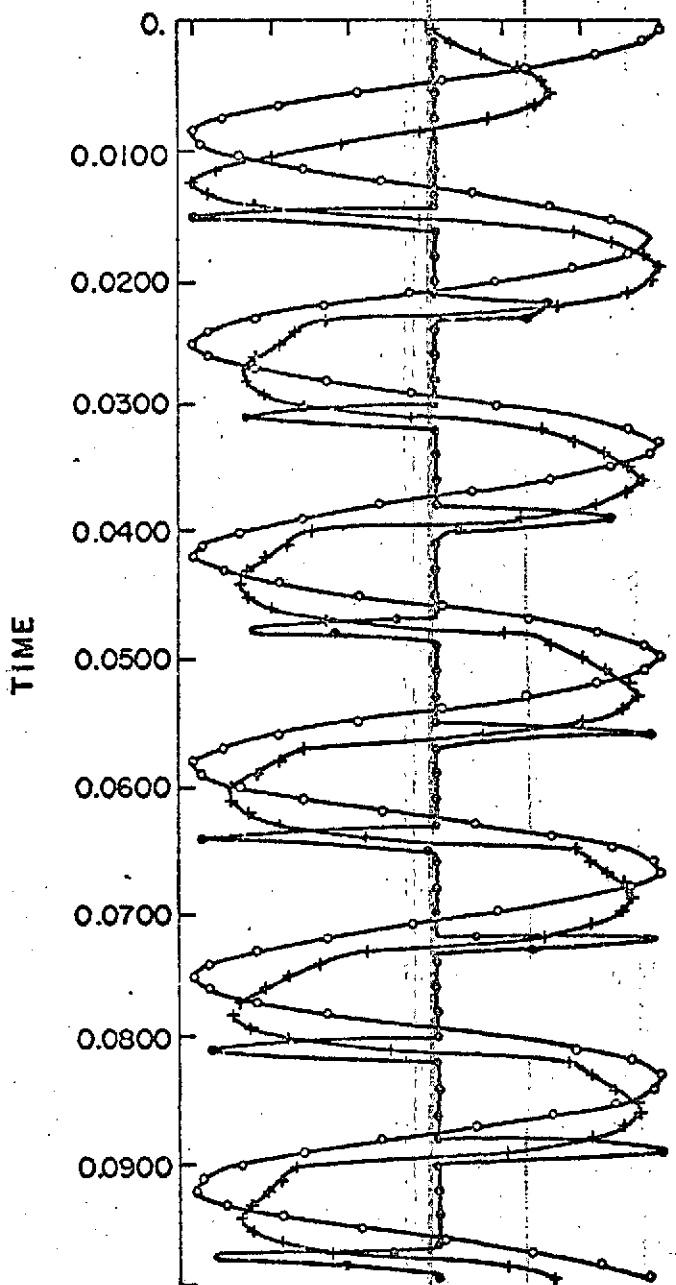
$|R| = V_{IN} = 149^V$

$\omega L < \frac{1}{\omega C}$

$|V_O| = 222^V$

Computer Plot 2

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	+ V_O	−253.22643	236.10812	8.15558
17	• i_{SR}	−3.17936	3.05205	0.10386
6	∘ V_I	−183.99999	184.00000	6.13333

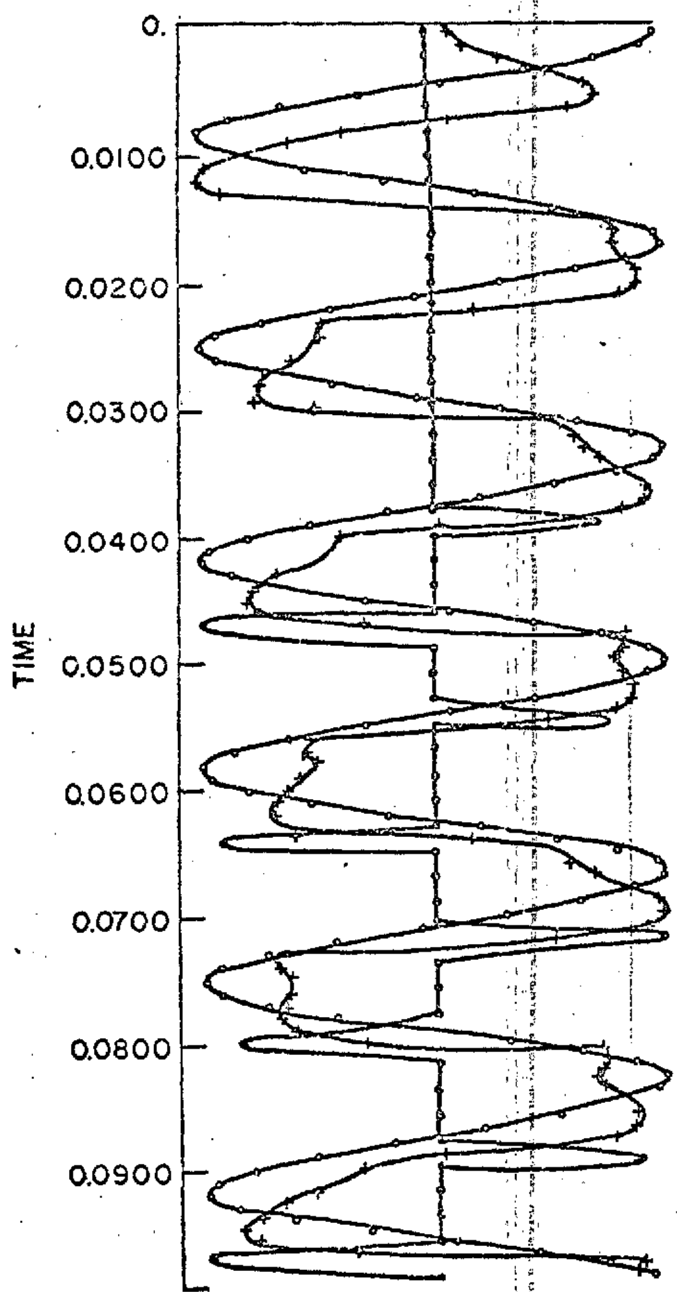


$|R| = V_{IN} = 184^V$
$\omega L < \frac{1}{\omega C}$
$|V_O| \doteq 220^V$

22

Computer Plot 3

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	$+V_o$	-278.60224	231.88214	8.50807
17	$\circ i_{SR}$	-5.13585	5.09260	0.17047
6	$\circ V_1$	-239.99999	240.00000	8.00000



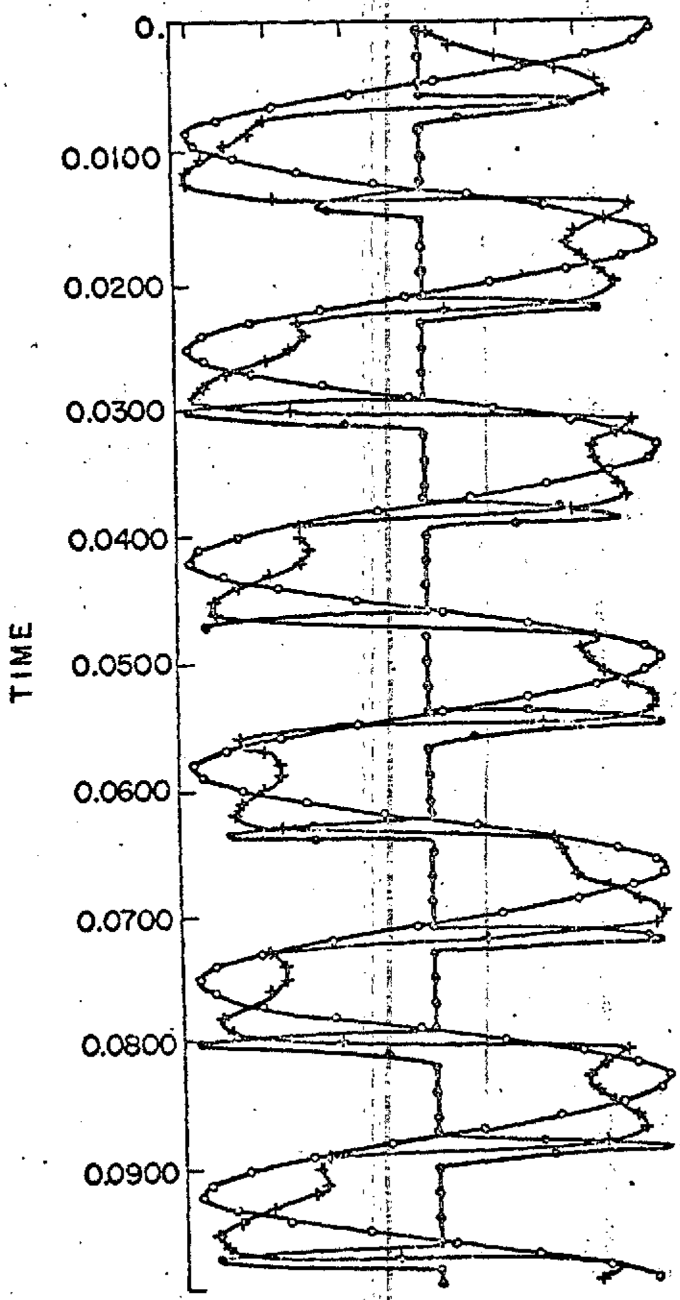
$V_{IN} = 240^V$

$\omega L < \frac{1}{\omega C}$

$|V_o| = 223^V$

Computer Plot 4

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	+ V_0	-252.90372	235.26656	8.13617
17	• i_{SR}	-5.47331	5.52829	0.18336
6	∘ V_I	-279.99999	280.00000	9.33333



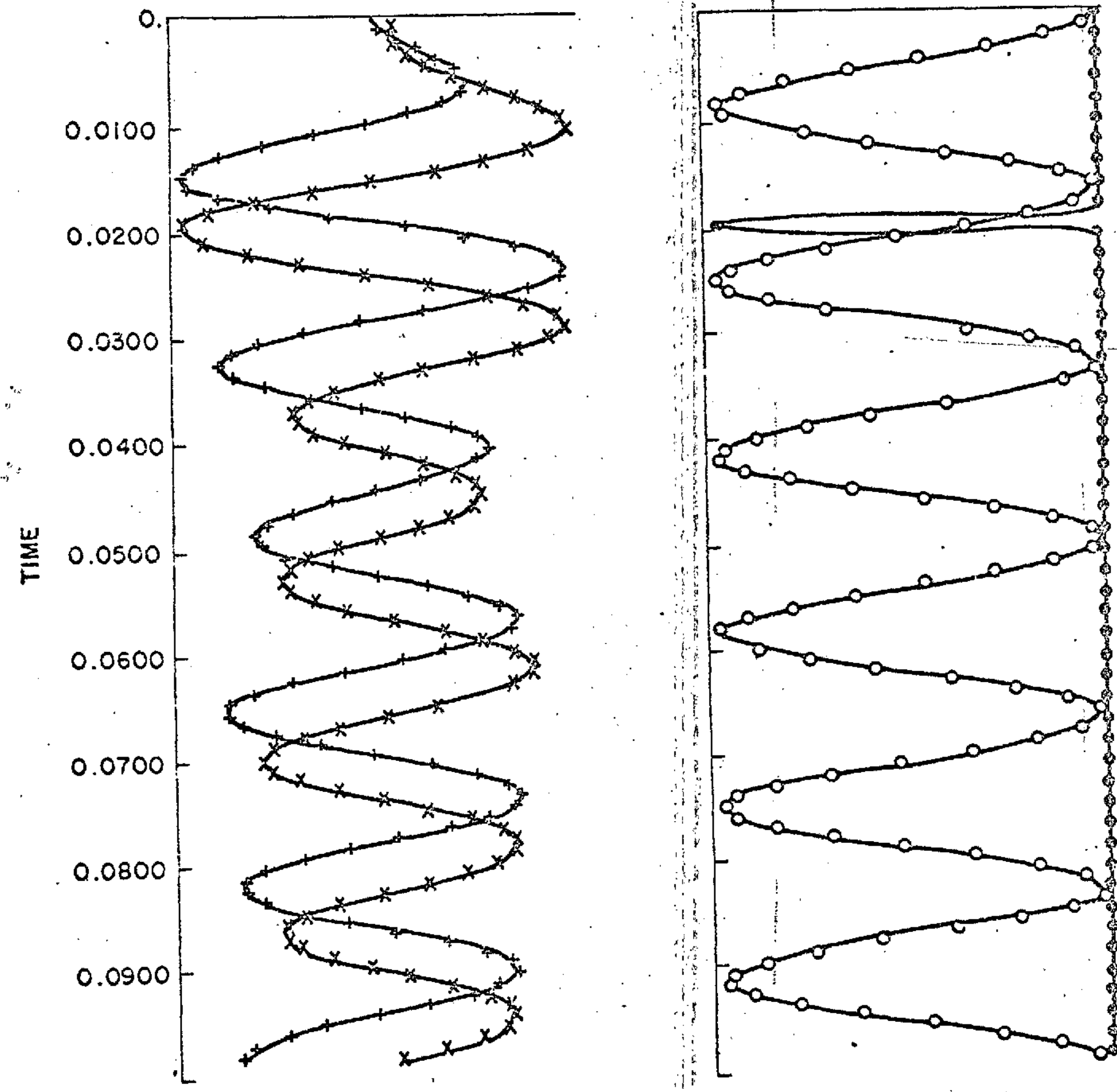
$|R| = V_{IN} = 280^V$

$\omega L < \frac{1}{\omega C}$

$|V_0| = 215^V$

Computer Plot 5

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	$+V_O$	-186.57184	189.69890	6.27118
17	$\odot\, i_{SR}$	-0.27906	0.	0.00465
15	$\times\, \phi$	-68558.11816	57466.74591	2100.41446
6	$\circ\, V_I$	-219.99999	220.00000	7.33333



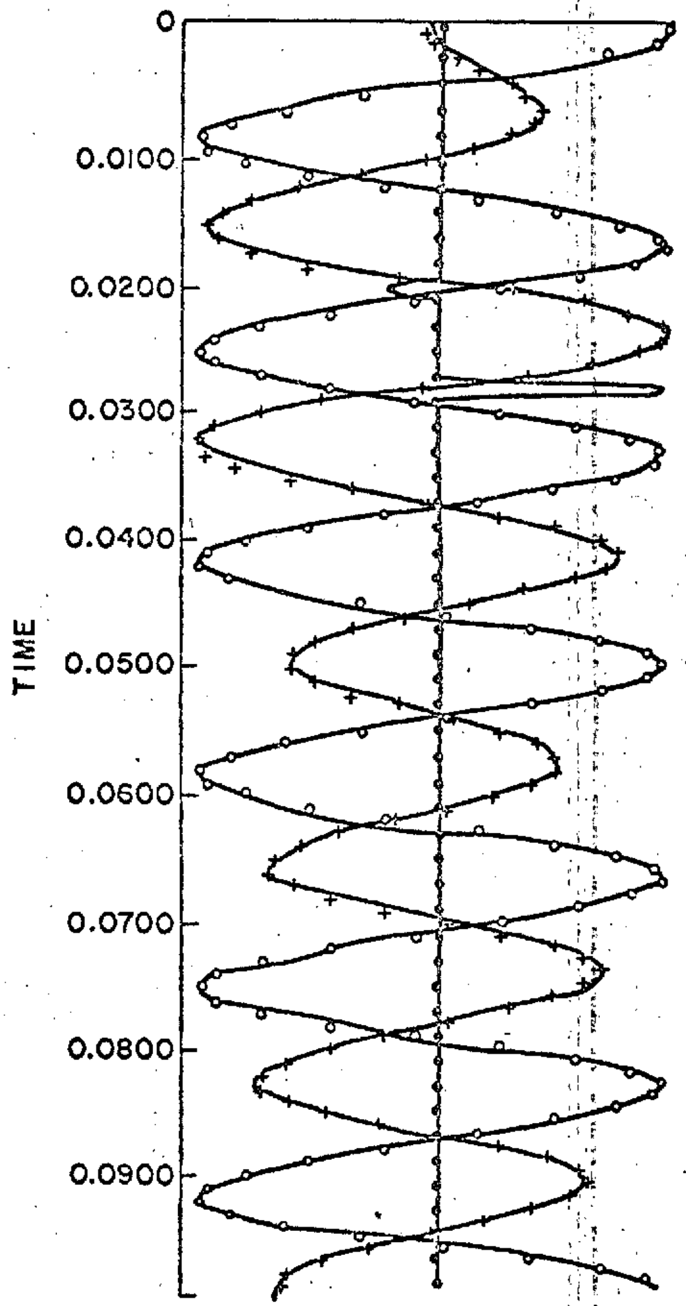
$V_{IN} = 220^V$

$\omega L > \frac{1}{\omega C}$

$|V_O| = 125^V$

Computer Plot 6

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	+ V_0	-210.68365	219.01813	7.16170
17	• i_{SR}	2.52219	2.34171	0.08107
6	• V_I	-239.99999	240.00000	8.00000



$V_{IN} = 240^V$

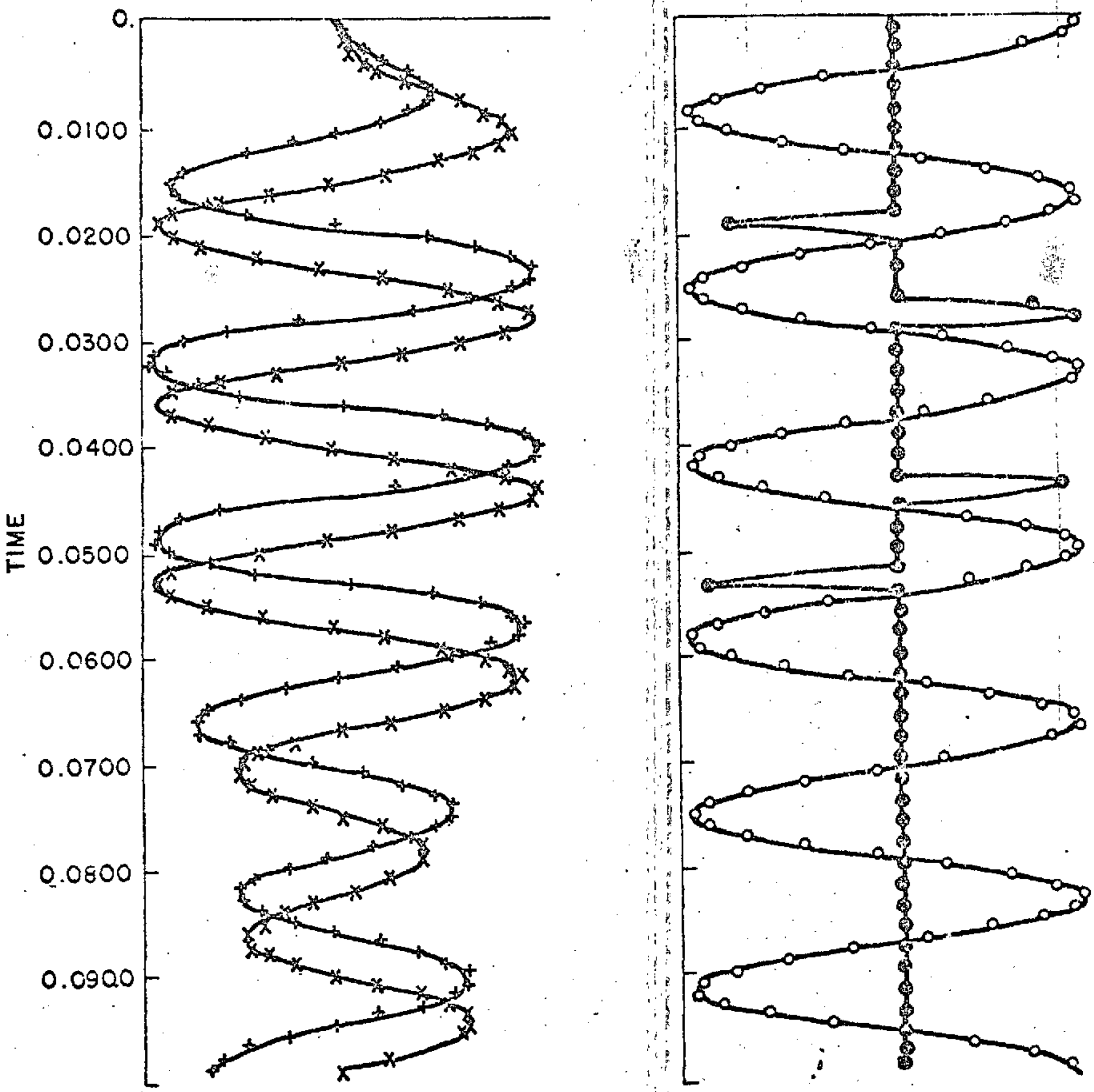
$WL > \frac{1}{WC}$

$|V_0| = 160^V$

26

Computer Plot 7

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	+ V_O	-231.28848	229.79834	7.68478
17	⊙ i_{SR}	-3.65168	3.13631	0.11313
15	x ϕ	-75303.36328	74272.61133	2492.93292
6	o V_I	-244.99999	245.00000	8.16667



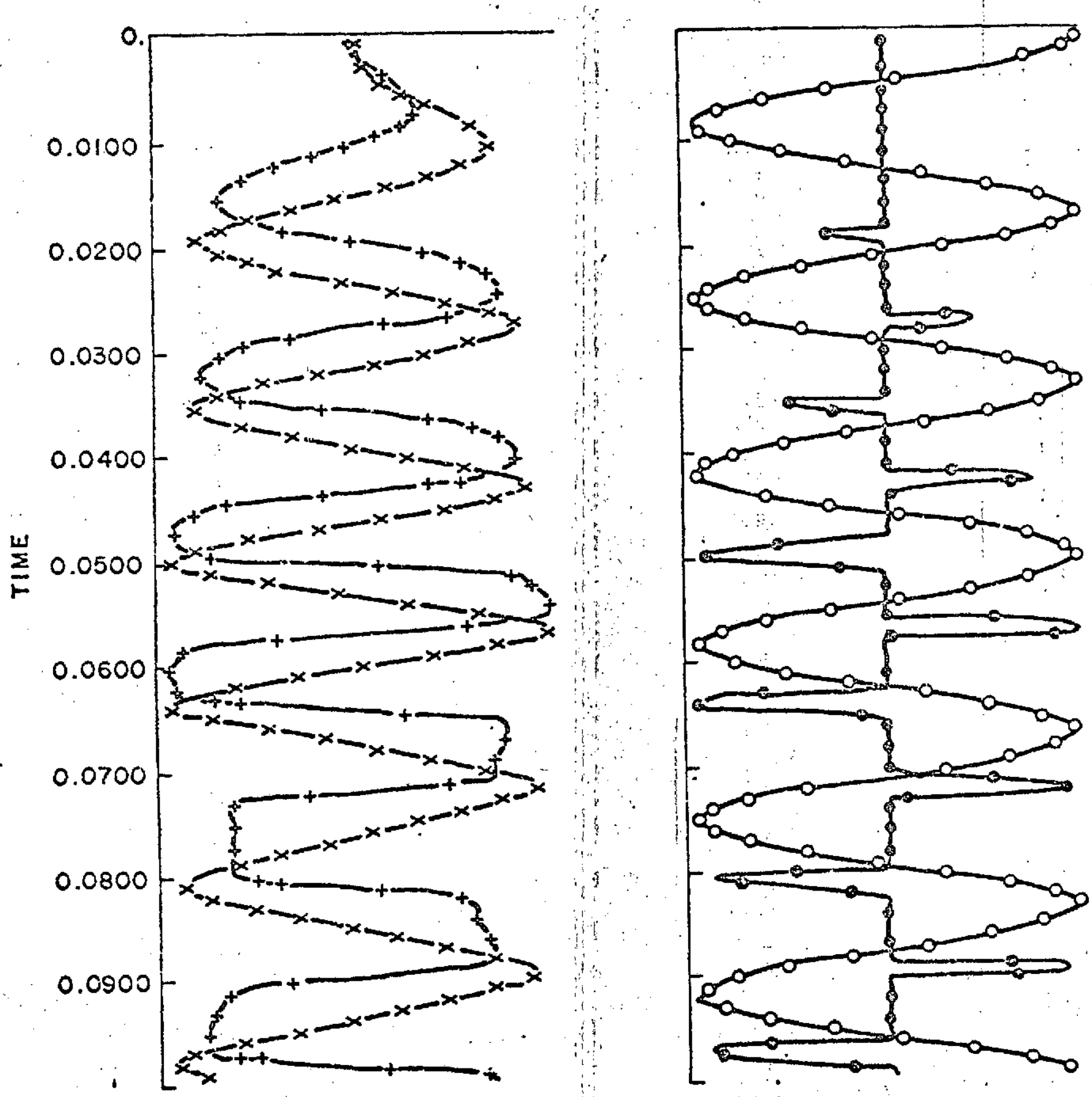
$V_{IN} = 245^V$

$\omega L > \frac{1}{\omega C}$

$|V_O| = 170^V$

Computer Plot 8

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	$+V_0$	−292.82857	315.13225	10.13268
17	$\odot\, i_{SR}$	−11.83963	12.17926	0.40031
15	$\times\, \phi$	−91679.25488	92358.51953	3067.29623
6	$\circ V_1$	−249.99999	250.00000	8.33333



$V_{IN} = 250^V$

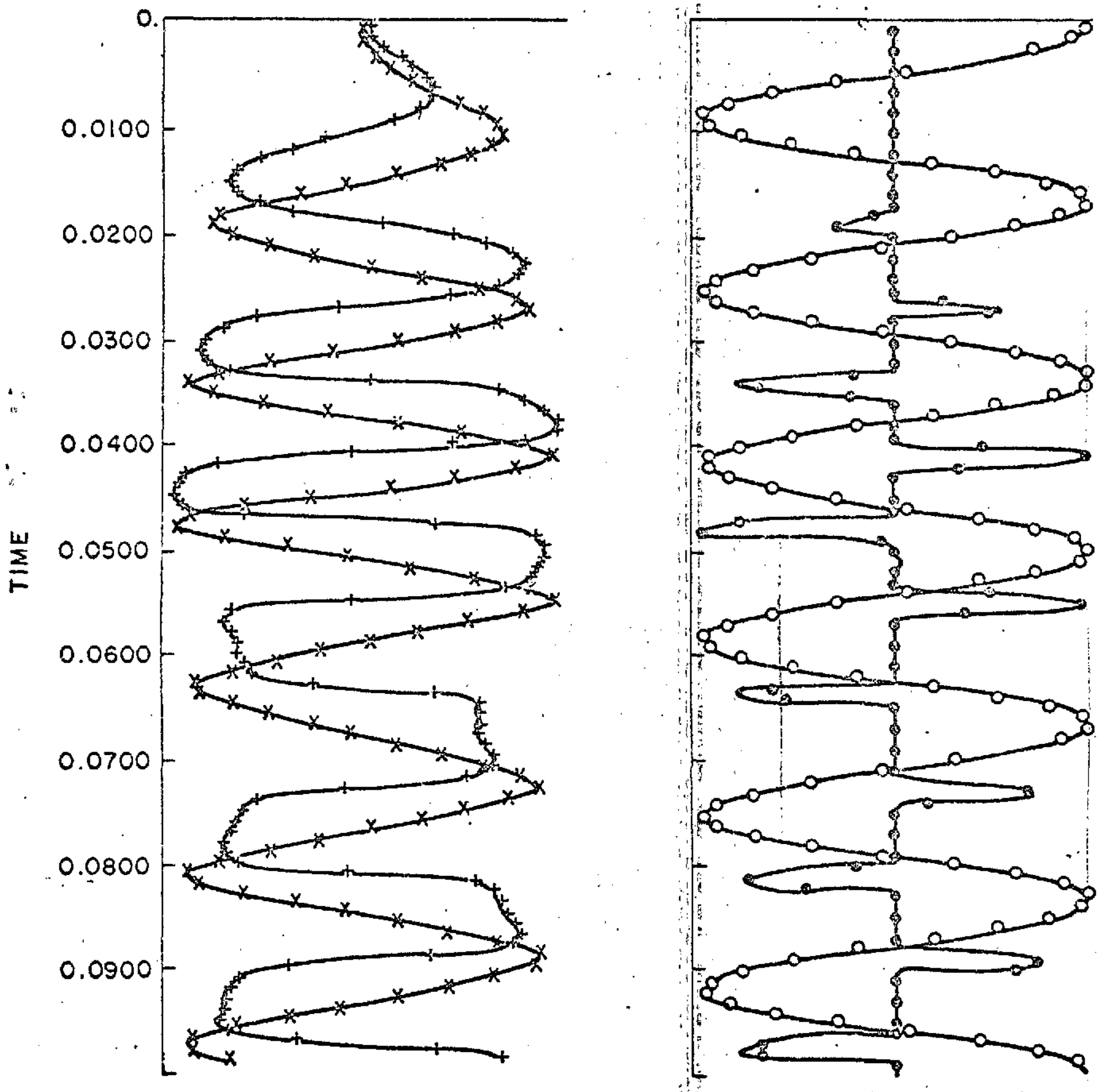
$\omega L > \frac{1}{\omega c}$

$|V_0| = 237^V$

THE JUMP HAS OCCURED!

Computer Plot 9

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	$+V_O$	315.32052	303.91003	10.32051
17	$\odot\, i_{SR}$	-12.90658	12.89402	0.43001
15	$\chi\, \phi$	-93813.15039	93788.03418	3126.68460
6	$\circ V_1$	-259.99999	260.00000	8.66667



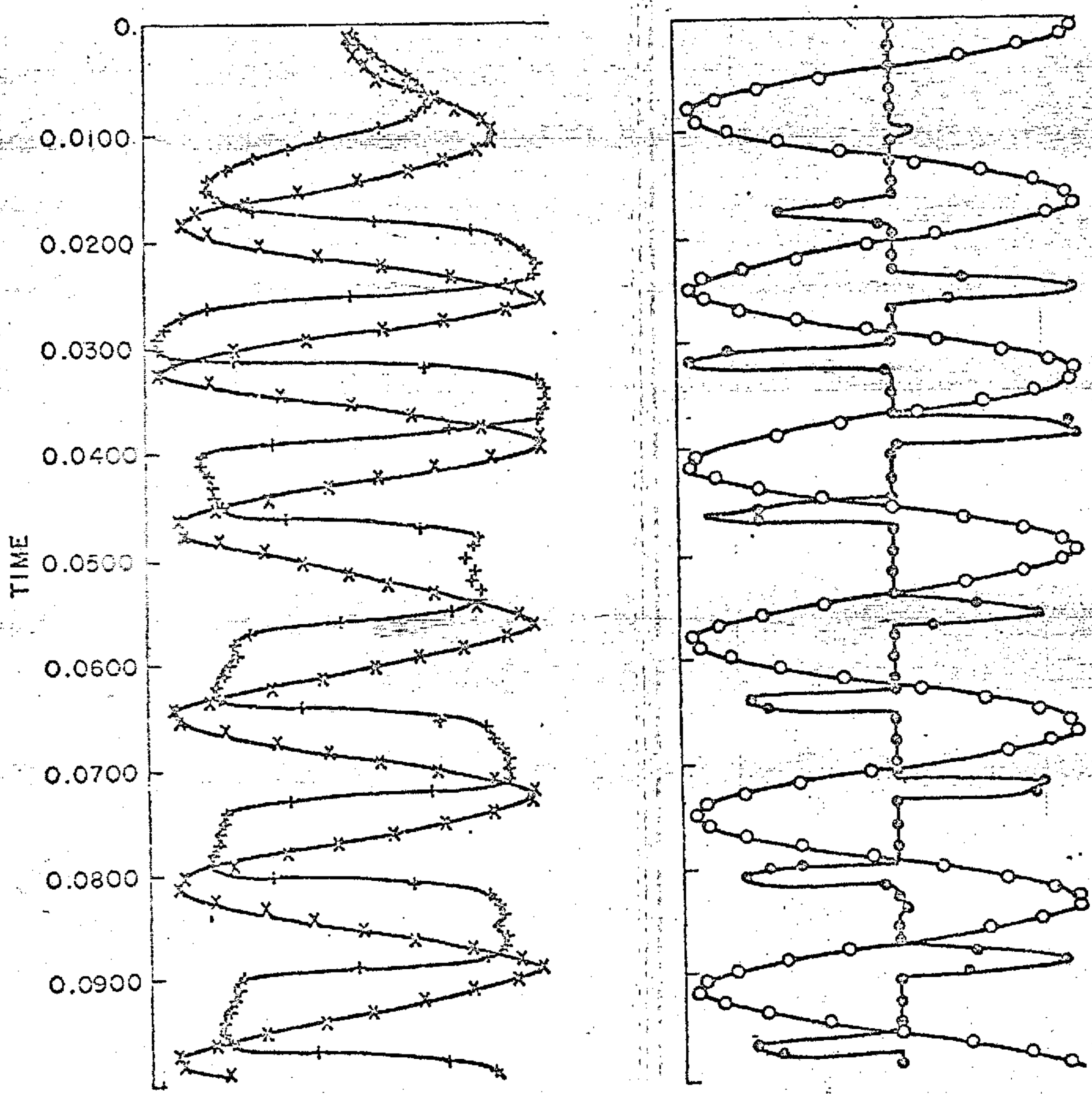
$V_{IN} = 260^V$

$\omega L > \frac{1}{\omega C}$

$|V_O| = 235^V$

Computer Plot 10

BLOCK	SYMBOL	LEFT	RIGHT	INCREMENT
12	+ V_0	-322.94545	-302.57053	10.42527
17.	o i_{SR}	-13.09731	11.61070	0.41180
15	x ϕ	-94194.61230	91221.39062	3090.26672
6	o V_I	-279.99999	280.00000	9.33333



$V_{IN} = 280^V$

$\omega L > \frac{1}{\omega C}$

$|V_O| = 235^V$